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An application of knowledge dependency to robot rod catching control

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Abstract. Depending on whether decision attribute contains missing attribute value or not, the concepts of knowledge dependency, partial dependency and dependency degree in incomplete information system are introduced to generalize relevant concepts defined in complete information systems. Relations between tolerance class and indispensable attribute and knowledge dependency are discussed. For complete dependency, reflectivity, transitivity, augmentation, decomposition rule and merge rule are still remained. Partial dependency degrees after transferring, augmenting and decomposing are not always preserved. The knowledge dependency in tolerance rough set model is applied to solve attribute reduction of an incomplete information decision table to implement robot catching control.

Key words. Attribute reduction, knowledge dependency, incomplete information system, robot control, rough set, tolerance relation..

1. Introduction

Through defining indiscernbility relation, rough set model can mine knowledge hidden in complete information systems well by using lower and upper approximations [1][2][3]. So the model is widely applied in many scientific fields such as decision making, data mining, pattern recognition, machine learning and etc. Because all attribute values for each sample or object are known in complete information systems, an indiscernbility relation or equivalence relation can be easily constructed to form equivalence classes as knowledge granules to study the information systems in rough set model. Owing to difficulties or other reasons such as data measuring or data acquiring limitation, some attribute values of sample objects are probably missing in the system. This means that incomplete information system (IIS) is always appeared in front of us. How to deal with incomplete information system if we face such kind of phenomenon?

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Currently, methods of processing incomplete information systems are divided into two categories. One is called reparation method or indirect method, in which missing data is replaced or filled in by some statistical value such as the mean or the most frequent value of the relevant attribute values to complete a system and then to process it.

The other is called direct method, in which missing data in incomplete information system remains unchanged. Related concepts in complete information system in rough set model are adequately expanded to incomplete one. For example, equivalence relation is substituted by tolerance relation, similarity relation; equivalence class by tolerance class or similarity class accordingly. In this way, many extended rough set models are built and used to dispose incomplete information system. Unlike indirect processing method for incomplete information system, the direct one avoids personal subjective perspectives. So the direct method now attracts interests from many scholars. Till now some real efficient research results have already obtained along this way. On missing value in incomplete information system, two different semantic explanations exist. One is that a missing value exist a value really but at present it is only missing. The other is that a missing value should remain absent and it is not allowed to compare with other values. On incomplete information systems, some expanded rough set models have already formed and studied in rough set area. Tolerance relation rough set model is put forward in [4]. Tolerance class is used to substitute equivalence class. Knowledge reduction is obtained by using generalized decision function. Non-symmetric similarity relation is introduced in [5][6]. Limited tolerant relation is proposed in [7] for making a little bit stricter condition on tolerance relation. Maximal consistent block technique for rule acquisition is suggested in [8][9]. Different information granules are used to obtain expanded rough set models from the view of general and complete covering [10]. Compatibility relation rough set model is discussed in [11]. Variable precision rough set models are discussed in many literatures [12]. Based on binary matrices algorithms to solve different lower and upper approximations are designed in [13]. Multi-granular rough set models are proposed and studied nowadays in[14]. These days expanded rough set models for incomplete information system have been explored by scientists from different subjects and become a hot topic[9][15]. Expanded rough set models also have been applied in different areas [16].

Out of comparing the difference of dependency and reduction between complete information system and the incomplete one[2], the present paper, under the first semantic explanation, mainly studies properties of tolerance class, attribute reduction, indispensable attribute, dispensable attribute and attribute core in incomplete information system. Relationships between tolerance class and indispensable attribute and dispensable attribute are discussed. Through defining knowledge dependency, partial knowledge dependency and knowledge dependency degree[17], regarding the situation of decision attribute having or not having missing value, it proves that knowledge dependency have the properties of reflexivity, transitivity, augmentation, decomposition and merge regularities in incomplete information system. Partial knowledge dependency degree after transferring, augmenting, decomposing possesses some regularities.

It uses knowledge dependency to find attribute reduction of an incomplete system and applies such technique to solve robot rod catching control. The work in the present paper helps distinguish some properties between incomplete information system and complete one, so it is meaningful.

2. Basic concepts

An incomplete information system is expressed by $IIS = (U, A, V, f)$ [4], where U is a finite non-empty set of objects; A is a finite non-empty set of condition attributes and decision attributes. For any $a \in A$ a function named f_a from U to V_a can be derived with a, where V_a is called the value range of attribute a. $f_a(x) = f(x, a)$. For an object, an attribute value may be lost or missing, In this case, it is called a null value (a null is denoted by*). For any $a \in A, * \in V_a$ may be true. $V = \cup V_a(a \in A)$ is called the set of all attribute values.

Definition 1. For $\forall c \subseteq A$ in the incomplete information system IIS, $SIM(c)$ is called a tolerance relation on U, where

$$SIM(c) = \{(x, y)|\forall a \in c(f_a(x) = f_a(y) \vee f_a(x) = * \vee f_a(y) = *)\}. \tag{1}$$

If $c = \{a\}, SIM(c) = SIM(\{a\})$ is abbreviated by $SIM(a)$.

$SIM(c)$ is reflexive and symmetric on U.

Definition 2. For $\forall p \subseteq A$,$S_p(x)$ is called a tolerance class, where

$$S_p(x) = \{y|(x, y) \in SIM(p)\}. \tag{2}$$

If $p = \{a\}, S_{\{a\}}(x)$ is denoted by $S_a(x)$ for short.

Definition 3. For $\forall p \subseteq A$,$U/SIM(p)$ is called a knowledge system on U where

$$U/SIM(p) = \{S_p(x)|x \in U\}. \tag{3}$$

$U/SIM(p)$ is a cover on U. Sometimes, it is denoted by U/P for short.

Definition 4. For any $X \subseteq U, p \subseteq A$,$P_-(X)$, and $P^-(X)$ are called the lower approximation and the upper approximation of X respectively, where

$$P_-(X) = \{y|y \in U, S_p(y) \subseteq X\}, \tag{4}$$

$$P^-(X) = \{y|y \in U, S_p(y) \cap X \neq \emptyset\}. \tag{5}$$

Definition 5. Let $p \subseteq A$,$p \neq \emptyset$.$a \in p$ is called dispensable in P if, and only if

$$SIM(p) = SIM(p - \{a\}). \tag{6}$$

Otherwise a is called indispensable.

Definition 6. p is called independent if, and only if each $a \in p$ is indispensable. Otherwise, p is called dependent.

Theorem 1. Let $p \subseteq A, a \in p$. If

$$S_a(x) = \cup S_{p-\{a\}}(y)(y \in S_a(x)), \tag{7}$$

for $\forall x \in U$, then a is dispensable in p, i.e.

$$SIM(p - \{a\}) = SIM(p). \tag{8}$$

3. Knowledge dependency in incomplete information system

Definition 7. For incomplete information system, let $a, b \in A. a \to b$ if, and only if for $\forall x_1, x_2 \in U, x_1 \neq x_2$, we have

$$\begin{aligned} &f_a(x_1) = f_a(x_2) \vee f_a(x_1) = * \vee f_a(x_2) = * \\ &\Rightarrow f_b(x_1) = f_b(x_2) \vee f_b(x_1) = * \vee f_b(x_2) = *. \end{aligned} \tag{9}$$

Theorem 2. Let $a, b \in A. a \to b$ if, and only if for $\forall x \in U$,

$$S_a(x) \subseteq S_b(x). \tag{10}$$

Definition 8. Let $p, q \subseteq A. p \to q$ if, and only if for any $a \in p$ and $b \in p$ it always holds that $a \to b$.

Theorem 3. Attribute set q is dependent on attribute set p, denoted by $p \to q$, if, and only if

$$SIM(p) \subseteq SIM(q). \tag{11}$$

When attribute set q is dependent on attribute set p, we also call that attribute set q is derived by attribute set p. This result has the similar form in complete information system. So it is a generalization from complete information system.

Theorem 4. Let $a \in p \subseteq A$. If $p - \{a\} \to p$, then a is dispensable in p.

Theorem 5. Let $a \in p \subseteq A$. If $p - \{a\} \to a$ and $a \to p - \{a\}$, then a is dispensable in p.

Theorem 6. Let $p, q \subseteq A. p \to q$ is equivalent to the following equation

$$SIM(p \cup q) = SIM(p). \tag{12}$$

Theorem 7. Let $p, q \subseteq A$. If $p \to q$ and $p \subseteq q$, then

$$POS_p(q) = \cup P\_(X)(X \in U/SIM(q)) = U. \tag{13}$$

4. The relation of knowledge dependency with tolerance class

Theorem 8. Let $a \in A, p \subseteq A$. If $p \to a$, then for $\forall x \in U$, we have

$$S_a(x) \subseteq \cup S_p(y)(y \in S_a(x)). \tag{14}$$

Theorem 9. For $\forall x \in U, p \subseteq A$, if

$$S_a(x) = \cup S_p(y)(y \in S_a(x)), \tag{15}$$

then

(i) $p \to a$, whether $a \in p$ or not.

(ii) a is dispensable in p, if $a \in p$.

Theorem 11. Let p, q, r, t be subsets of A. Following laws can be proved.

(i) Reflexivity law: if $q \subseteq p \subseteq A$, then $p \to q$.

(ii) Transitivity law: if $p \to q$, and $q \to r$, then $p \to r$.

(iii) Left merge law: if $p \to r$, and $q \to r$, then $p \cup q \to r$.

(iv) Decomposition law: if $p \to q \cup r$, then $p \to q$ and $p \to r$.

(v) Pseudo Transitivity law: if $p \to q$ and $q \cup r \to t$, then $p \cup r \to t$.

(vi) Merge law: if $p \to q$ and $r \to t$, then $p \cup r \to q \cup t$.

(vii) Augmentation law: if $p \to q$ and $p \subseteq r$, then $r \to q$.

The decomposition law (iv) can be rewritten in the following equivalent form:

(iv') Decomposition law: if $p \to q$ and $r \subseteq q$, then $p \to r$.

The augmentation (vii) can be rewritten in the following equivalent form:

(vii') Augmentation law: if $p \to q, r \subseteq A$, then $p \cup r \to q \cup r$.

Definition 9. Let $p, q \subseteq A, U = \{x_1, x_2, ..., x_n\}$ and n be a positive constant integer. q is called partially dependent on p with degree k, denoted by $p \to_k q$, where

$$k = k(p, q) = \sum{}_{i=1}^{n} |S_p(x_i) \cap S_q(x_i)| / \sum{}_{i=1}^{n} |S_p(x_i)|. \tag{16}$$

It is obvious that $0 \leq k \leq 1$. Because for any $p, q \subseteq A$, we have $x \in S_p(x) \neq \emptyset$, and $x \in S_q(x) \neq \emptyset$, and

$$x \in S_p(x) \cap S_q(x) \neq \emptyset. \tag{17}$$

Thus, $0 < k \leq 1$ in general. $k = 0$ only if $q = \emptyset$.

If q is completely dependent on p, i.e., $p \to q$, then according to the computation of dependency degree it must have $p \to_k q$, where $k = 1. p \to_1 q$ and $p \to q$ are of the same meaning.

Theorem 12 For $\forall p, q \subseteq A, p \to q$ if, and only if for $\forall x \in U$,

$$S_p(x) \subseteq S_q(x) \tag{18}$$

5. Properties of partial knowledge dependency

Theorems in this section are all related to the concept of knowledge dependency and knowledge dependency degree. That is, knowledge dependency between conditional attribute subsets is considered. Knowledge dependency between conditional attribute subset and decision attribute subset can be viewed as special case of knowledge dependency between conditional attribute subsets.

Theorem 13. For $\forall p, q, r \subseteq A, U = \{x_1, x_2, ..., x_n\}$, we have

(i) if $p \to q, q{\to_{k_1}}r$, then $p{\to_{k_2}}r, k_2 \leq \alpha k_1$, where

$$\alpha = \sum\nolimits_{i=1}^{n}|S_q(x_i)|/\sum\nolimits_{i=1}^{n}|S_p(x_i)| \geq 1 \tag{19}$$

(ii) if $p{\to_{k_1}}q, q \to r$, then $p{\to_{k_2}}r$ and $k_2 \geq k_1$.

Theorem 14. For $\forall p, q, r \subseteq A$, we have
if $p{\to_{k_1}}q$, then $p \cup r{\to_{k_2}}q, \alpha k_1 \geq k_2$, where

$$\alpha = \sum\nolimits_{i=1}^{n}|S_p(x_i)|/\sum\nolimits_{i=1}^{n}|S_{p\cup r}(x_i)| \geq 1 \tag{20}$$

(ii) if $p{\to_{k_1}}q$ and $p{\to_{k_2}}q \cup r$, then $k_2 \leq k_1$.

Theorem 15. For $\forall p, q, r \subseteq A$, if $q \subseteq p, r{\to_{k_1}}p$, then $r{\to_{k_2}}q$, and $k_2 \geq k_1$.

Theorem 16. For $\forall p, q, r \subseteq A$, if $p \cup r{\to_{k_1}}q$, then $p{\to_{k_2}}q, r{\to_{k_3}}q$, and $k_1 \leq \min\{\alpha k_2, \beta k_3\}$, where

$$\alpha = \sum\nolimits_{i=1}^{n}|S_p(x_i)|/\sum\nolimits_{i=1}^{n}|S_{p\cup r}(x_i)| \geq 1, \tag{21}$$

$$\beta = \sum\nolimits_{i=1}^{n}|S_r(x_i)|/\sum\nolimits_{i=1}^{n}|S_{p\cup r}(x_i)| \geq 1. \tag{22}$$

This theorem reveals some regularities of knowledge dependency degree on partial knowledge and the entire knowledge.

In addition, the following result can be obtained.

Theorem 17. For $\forall p, q, r \subseteq A$, if $p \to q, r{\to_{k_1}}q$, then $r{\to_{k_2}}p, k_2 \leq k_1$.

Theorem 18. For $\forall p, q, r \subseteq A$, if $p \to q, p{\to_{k_1}}r$, then $q{\to_{k_2}}r, k_1 \leq \alpha k_2$,, where

$$\alpha = \sum\nolimits_{i=1}^{n}|S_q(x_i)|/\sum\nolimits_{i=1}^{n}|S_p(x_i)| \geq 1 \tag{23}$$

All theorems in this paper are proved by us. In order to save space the proofs of them are all omitted.

6. An application to robot rod catching control

A robot rod catching control decision system is shown in Table 1.

The robot has 6 situation state attributes: $a, b, c, d, e, f.A = \{a, b, c, d, e\}.a, b, c, d, e$ are all condition attributes, f is the decision attribute. They have meanings as follows: a represents that the rod is at the central point or not. 1 means at, 0 means not at. b represents the robot is facing at the rod or not, 1 means facing at, 0 means not facing at. c represents that robot is at the central line of the rod or not, 1 means at, 0 means not at. d represents that the robot is facing forward to the central line of the rod or not, 1 means facing to, 0 means not facing to. e denotes that the robot catches or not. 1 means cached, 0 means not. f represents the behavior of the robot, 1 means rotating, 2 means move forward, 3 means catching, 4 means stop. * in the

Table 1. An IIS of Robot catching rod situations

U	a	b	c	d	e	f
1	0	0	*	*	0	1
2	0	*	1	0	*	1
3	1	0	*	*	0	1
4	*	*	0	1	0	2
5	0	*	1	*	0	2
6	1	*	1	0	*	3
7	1	1	*	*	*	4

Table 2. A reduct of Table 1

U	a	b	c	d	f
1	0	0	*	*	1
2	0	*	1	0	1
3	1	0	*	*	1
4	*	*	0	1	2
5	0	*	1	*	2
6	1	*	1	0	3
7	1	1	*	*	4

table means the relevant attribute value is not determine, may missing out of some reasons. So it is an incomplete information system.

For the incomplete information system, we can build a tolerance relation according to rough set model. We also can check the dependency between attribute subsets and calculate dependency degree according to the above research work. Since

$$SIM(A - \{e\} \cup \{f\}) \subseteq SIM(A \cup \{f\}), \tag{24}$$

i.e.

$$A - \{e\} \cup \{f\} \rightarrow A \cup \{f\}, \tag{25}$$

and

$$k(A - \{e\}k(A - \{e\} \cup \{f\}, A \cup \{f\} = 1, \tag{26}$$

we know that e is a dispensable attribute in A. We can check that only e is dispensable attribute in A. e is a redundant attribute. Other attributes are all indispensable. So $\{a, b, c, d\}$ is a unique reduction of the system, see Table 2. That means the control of the robot can only depend on attributes $\{a, b, c, d\}$. We may pay much attention to the control of them.

7. Conclusion

Along with the concepts of dependency defined in complete information system, the present discusses the dependency in incomplete information system, especially defines partial dependency and partial dependency degree regarding decision attribute contains missing value or not. The properties of indispensable attribute and dispensable attribute in attribute set with the tolerance is explored in tolerance rough set model. Some necessary and/or sufficient conditions for dispensable attribute are obtained. It proves that knowledge dependency possesses reflexivity, transitivity, augmentation, and decomposition laws in incomplete information system. After transferring, augmentation, and decomposition, partial knowledge dependency degree conform some special laws illustrated in theorems. Based on knowledge dependency or knowledge dependency degree, an attribute reduction of an incomplete information system can be solved. Used to solve the reduction of robot rod catching problem, the method achieves at the same reduction result. It proved that the correctness of our research method and reduction are efficient.

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